Opportunistic Cooperation with Receiver-Based Ratio Combining Strategy

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Abstract. In cooperative wireless communication systems, many combining techniques could be employed at the receiver, such as maximal ratio combining (MRC), equal gain combining (EGC), etc. To address the effect of receiver diversity combining on optimum energy allocation, we analyze the problem of minimizing average total transmit energy under a SNR constraint when different ratio combining methods are utilized at destination. For maximal ratio combining (MRC), based on the explicit analytical solution an asymptotic solution for normalized optimum total energy in terms of μ and η was derived in the high-SNR scenario. For fixed ratio combining (FRC), we find that there does not exist an explicit analytical solution to the optimum energy allocation problem. However, the convexity proof for the energy function provides a way of using numerical convex optimization methods to find the unique solution. Our results also show that, while direct transmission ($\mathcal{E}_r^* = 0$) is optimum for certain channel states when the destination uses MRC, the relay should always transmit, i.e. $\mathcal{E}_r^* > 0$ for all channel states, when the combining ratio β is a fixed number.

1 Introduction

In cooperative wireless communication, each user is assumed to transmit data as well as act as a cooperative agent for another user. The transmitters or receivers can collectively act as an antenna array and create a virtual or distributed multiple-input multiple-output (MIMO) system. The basic ideas behind cooperative communication can be traced back to the work of Cover and El Gamal on the information theoretic properties of the relay channel [1]. However, the earliest work specifically on user cooperation is due to Sedonaris et al. in [2]-[3] for cellular networks and Laneman et al. in [4]-[5] for *ad hoc* networks.

It has been shown that the cooperative transmission strategy provides powerful benefits of multi-antenna systems without the need for physical arrays, e.g. an increased capacity, a robustness to fading and reduced outage probability. Recent results in implementation of different cooperative signaling methods such

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as amplify-and-forward [4] and decode-and-forward [6]-[7], indicate that cooperative communication has a promising future. These results also demonstrated that while knowledge of channel state information at the transmitters (CSIT) is beneficial, it is not necessary to achieve significant gains in energy efficiency with respect to direct (non-cooperative) transmission.

While recent work in this area has focused on the goal of minimizing BER, minimizing total power to a rate constraint, minimizing total power subject to fixed SNR and outage probability constraints, the problem of how the diversity combining methods affect the optimum energy allocation has not been fully investigated.

In this paper, we consider the problem of optimum energy allocation and weighted total energy minimization under SNR constraint in two scenarios: (i) $\beta = \beta_{\mathsf{mrc}}$, i.e. maximal ratio combining (MRC) is used at the receiver and (ii) β is a fixed number, i.e. fixed ratio combining (FRC) is used at the receiver. In both cases, we derive the optimum opportunistic energy allocation strategies and explicitly describe the set of channel conditions under which the objective can be realized. Our analysis shows that, when MRC is utilized at destination, cooperative transmission is more energy efficient than direct transmission except when the relay-destination channel is not advantaged. The asymptotic solution we derived for the high-SNR scenario can best illustrate this. We also show that, while direct transmission ($\mathcal{E}_r^* = 0$) is optimum for certain channel states when the destination uses MRC, the relay should always transmit when fixed ratio combining (FRC) is utilized at destination, i.e. $\mathcal{E}_r^* > 0$ for all channel states. The impact of channel state information on AF cooperative transmission using MRC and EGC has been studied in [8] and [9], respectively, the intuitive results in this paper could be regarded as an extension of prior works.

2 System Model

To facilitate analysis, we consider the same system model as in [8] (Figure 1). In the network, each source is both a user and a relay, one source communicates directly to a destination and another source acts as a relay under certain channel conditions. The channels in the system are all assumed to be frequency non-selective and the channel magnitudes $|g_s|$, $|g_r|$, and $|h|$ are assumed to be independent Rayleigh distributed random variables. We also assume the channels stay roughly constant for several timeslots, i.e., in the process of cooperation.

In this paper, we use Amplify-and-forward as our signaling method in cooperative communication sytem. Amplify-and-forward is a simple method that lends itself to analysis, and thus has been very useful in furthering our understanding of cooperative communication systems. This method was proposed and analyzed by Laneman et al. [4]. It has been shown that for the two-source case, this method achieves diversity order of two, which is the best possible outcome at high SNR. In AF, each source receives a noisy version of the signal transmitted by its partner (relay). The relay then amplifies and retransmits this noisy

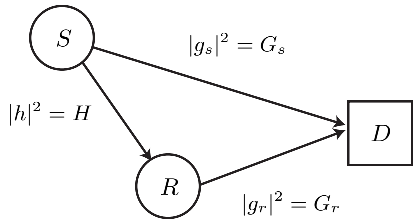

Fig. 1. System model

version. The destination combines the information sent by the source and relay, and makes a final decision on the transmitted bit.

– In the first timeslot, the source transmits the symbol x to the destination. The signals received by the destination and relay in this timeslot are as follows

$$y_{sd} = |g_s|a_s x + w_{sd}$$
$$y_{sr} = |h|a_s x + w_{sr},$$

where a_s is the amplitude of the source's transmission and w_{sd} and w_{sr} are additive white Gaussian noise at the receivers of the destination and relay , respectively.
– In the second timeslot, the relay retransmits the signal that it observed in the first timeslot to the destination. The signal received by the destination in this slot is

$$\begin{aligned} y_{rd} &= |g_r|a_r y_{sr} + w_{rd} \\ &= |g_r|a_r|h|a_s x + |g_r|a_r w_{sr} + w_{rd} \end{aligned}$$

where a_r is the amplitude of the relay's transmission and w_{rd} denotes the receiver noise at the destination in the second timeslot.
– The destination makes a final decision on x based on the observations in the two timeslots

$$y_d = \beta_1 y_{sd} + \beta_2 y_{rd}$$

where β_1 and β_2 are nonnegative combining ratios.

3 SNR Analysis

We model AWGN as independent normal random variables with zero mean and unit variance. The instantaneous SNR of the final decision can be written as

$$\mathsf{SNR}(\beta_1, \beta_2) = \frac{\big(\beta_1|g_s| + \beta_2|g_r|a_r|h|\big)^2 a_s^2 \mathrm{E}[x^H x]}{\beta_1^2 + \beta_2^2\big(|g_r|^2 a_r^2 + 1]\big)} \quad (1)$$

By setting $\beta = \beta_1/\beta_2$ and plugging $G_s = |g_s|^2$, $G_r = |g_r|^2$, $H = |h|^2$, (1) can be rewritten as

$$\mathsf{SNR} \quad = \quad \frac{\beta^2 G_s \mathcal{E}_s (H\mathcal{E}_s + 1) + G_r \mathcal{E}_r H \mathcal{E}_s + 2\beta \mathcal{E}_s \sqrt{G_r \mathcal{E}_r G_s H (H\mathcal{E}_s + 1)}}{G_r \mathcal{E}_r + (\beta^2 + 1)(H\mathcal{E}_s + 1)} \tag{2}$$

where $\mathcal{E}_s = a_s^2 \mathrm{E}[x^H x]$ and $\mathcal{E}_r = a_r^2 (H\mathcal{E}_s + 1)$.

Note that the relay transmission energy is conditioned on $H\mathcal{E}_s$ and includes both a signal component and a noise component. The noise component is a consequence of the fact that the relay transmission is simply an amplified copy of the noisy signal received in the first timeslot.

When the destination has full access to the channel state information (CSI) and transmit energies, maximal ratio combining (MRC) can be used to maximize the SNR of the decision statistic. The resulting instantaneous SNR at the destination, after MRC, can be expressed as [8]

$$\mathsf{SNR}_{\mathsf{mrc}} = G_s \mathcal{E}_s + \frac{G_r \mathcal{E}_r H \mathcal{E}_s}{G_r \mathcal{E}_r + H\mathcal{E}_s + 1}. \tag{3}$$

where

$$\beta_{\mathsf{mrc}}^2 = \frac{G_s \left((H\mathcal{E}_s + 1) + G_r \mathcal{E}_r\right)^2}{G_r H \mathcal{E}_r (H\mathcal{E}_s + 1)}.$$

Note that the first part of (3) is the SNR of direct transmission.

When the destination does not have access to the channel state, equal gain combining (EGC) can be used (i.e. $\beta_{\mathsf{egc}} = 1$). The resulting instantaneous SNR at the destination, after EGC, can be expressed as [9]

$$\mathsf{SNR}_{\mathsf{egc}} \quad = \quad \frac{G_s \mathcal{E}_s}{2} + \frac{G_r \mathcal{E}_r \mathcal{E}_s \left(H - G_s/2\right) + 2\mathcal{E}_s \sqrt{G_r \mathcal{E}_r G_s H (H\mathcal{E}_s + 1)}}{G_r \mathcal{E}_r + 2(H\mathcal{E}_s + 1)}. \tag{4}$$

In this paper, to establish a framework for optimum energy allocation, we define

$$\mathcal{E}_{\mathsf{tot}} = \mathcal{E}_s + \alpha \mathcal{E}_r \tag{5}$$

as the *weighted total transmission energy* used in the cooperative transmission interval. The parameter $\alpha \geq 0$ allows for a weighting of the cost of the relay's energy relative to the cost of the source's energy. The following sections derive the optimum energy allocation strategies for an AF cooperative transmission under MRC and FRC using the weighted total transmission energy metric (5).

4 Optimum Energy Allocation

In this section, for a given channel state $\mathbf{s} = \{G_s, G_r, H\}$, we consider the problem finding the optimum energy allocation $\{\mathcal{E}_s^*, \mathcal{E}_r^*\}$ that minimizes the weighted total energy under a minimum SNR constraint ρ, i.e.,

$$\mathcal{E}_{\mathsf{tot}}^* = \min_{\{\mathcal{E}_s, \mathcal{E}_r\} \in \mathcal{B}} \mathcal{E}_{\mathsf{tot}} \tag{6}$$

where $\mathcal{B}$ is the set of energy allocations satisfying $\mathcal{E}_s \geq 0$, $\mathcal{E}_r \geq 0$, and the SNR constraint $\mathsf{SNR}(\mathbf{s}, \beta, \mathcal{E}_s, \mathcal{E}_r) \geq \rho$.

The SNR of the sources' information at the destination is determined not only by the channel states and the transmission energies but also by how the destination forms its decision statistic from the received source and relay transmissions. In the following sections, we first analyze the energy minimization problem when MRC technique is utilized at the destination, where the destination has full access to the channel states and transmit energies of both sources in both timeslots. In this case, β is a function of channel states and transmit power. In the second part of this section, we will discuss another situation, when fixed ratio combining (FRC) is utilized at the destination, i.e., β is a fixed number.

4.1 Maximal Ratio Combining

To facilitate analysis of (6) in the case of a destination using MRC, we define two non-negative quantities

$$\mu := \frac{\alpha G_s}{G_r}\left(1 + \frac{G_s}{H\rho}\right) \tag{7}$$

and

$$\eta := \frac{H}{H + G_s}. \tag{8}$$

The explicit solution to the total energy minimization problem for a destination using MRC is given in the following proposition.

Proposition 1. *When $\beta = \beta_{\mathsf{mrc}}$, the normalized minimum weighted total energy $\frac{\mathcal{E}_{\mathsf{tot}}^*}{\rho G_s^{-1}}$ can be expressed as*

$$\begin{cases} \frac{\left(\sqrt{\rho G_s(G_r(G_s+H)-\alpha G_s H)}+\sqrt{\alpha G_s H(G_s+H+\rho H)}\right)^2-\alpha G_s(G_s+H)^2}{\rho G_r (G_s+H)^2} & 0 \leq \mu < 1, \\ 1 & \mu \geq 1. \end{cases} \tag{9}$$

The proof of Proposition 1 is provided in Appendix A.

We note that when the SNR constraint $\rho \to \infty$, $\mu \approx \alpha G_s / G_r$ can be considered an indicator of source or relay channel advantage, i.e. $\mu > 1$ indicates that the source has an advantaged channel to the destination and $\mu < 1$ indicates that the relay is advantaged. Similarly, η can be considered an indicator of source-relay or source advantage. When H is large with respect to G_s, the quantity $\eta \approx 1$, which means the source and relay are much closer in proximity than the source and destination.

Without loss of generality, we consider the problem in high-SNR scenario. When $\rho \to \infty$, the normalized minimum weighted total energy can be expressed in terms of μ and η as

$$\mathcal{E}_{\mathsf{tot}}^*/\rho G_s^{-1} = \begin{cases} \left((\eta^2\mu)^{\frac{1}{2}} + [(1-\eta)(1-\eta\mu)]^{\frac{1}{2}}\right)^2 & \text{when } 0 \le \mu < 1, \\ 1 & \text{when } \mu \ge 1. \end{cases} \quad (10)$$

We can also define the total energy gain of optimum cooperative transmission as the ratio of the $\mathcal{E}_{\mathsf{tot}}$ achieved with direct transmission, i.e. $\frac{\rho}{G_s}$ to the $\mathcal{E}_{\mathsf{tot}}^*$ achieved with optimum AF cooperative transmission.

Similarly, we can show that the asymptotic solution for normalized optimum source energy in the high-SNR scenario can be expressed as

$$\mathcal{E}_s^*/\mathcal{E}_{\mathsf{tot}}^* = \begin{cases} (\frac{1-\eta}{1-\eta\mu})^{\frac{1}{2}} \left((\eta^2\mu)^{\frac{1}{2}} + [(1-\eta)(1-\eta\mu)]^{\frac{1}{2}}\right)^{-1} & \text{when } 0 \le \mu < 1, \\ 1 & \text{when } \mu \ge 1. \end{cases} \quad (11)$$

4.2 Fixed Ratio Combining

This section analyzes the scenario when FRC is used at the destination (β is a fixed number), i.e. it is not dependent on the channel states and transmit power. Note that equal gain combining (EGC) can be considered as a special case of FRC where $\beta_{\mathsf{egc}} = 1$.

The relay node energy $\mathcal{E}_r$ can be written as a function of ρ and $\mathcal{E}_s$ by solving (2) for $\mathcal{E}_r$ when $\mathsf{SNR}(\beta) = \rho$. The solution yields two roots for $\mathcal{E}_r$. When $\mathcal{E}_r = 0$, by solving the equation $\mathsf{SNR}(\beta) = \rho$ we have $\mathcal{E}_s = \frac{(\beta^2+1)\rho}{\beta^2 G_s}$. The correct root should satisfy this condition and can be written as

$$\mathcal{E}_{\mathsf{r}} = \frac{(H\mathcal{E}_s+1)(\beta^2 G_s H \mathcal{E}_s^2 + (\beta^2+1)\rho H \mathcal{E}_s + \beta^2 G_s \mathcal{E}_s \rho - (\beta^2+1)\rho^2)}{G_r(\rho - H\mathcal{E}_s)^2} - \frac{2\beta\mathcal{E}_s\sqrt{G_s H \rho[(\beta^2+1)H\mathcal{E}_s + \beta^2 G_s \mathcal{E}_s - (\beta^2+1)\rho]}}{G_r(\rho - H\mathcal{E}_s)^2} \quad (12)$$

The admissible range of instantaneous energy allocations that satisfy $\mathsf{SNR}(\beta) = \rho$ can be described as the region in $\mathbb{R}^2$ where $\mathcal{E}_r \ge 0$ and $\frac{(\beta^2+1)\rho}{(\beta^2+1)H+\beta^2 G_s} \le \mathcal{E}_s \le \frac{(\beta^2+1)\rho}{\beta^2 G_s}$. The case $\mathcal{E}_r = 0$ establishes the upper limit on the interval of admissible solutions for $\mathcal{E}_s$. The lower limit on the interval is established by the requirement for total energy to be a real-valued quantity. The square root in the numerator of (12) reveals that $\mathcal{E}_r \in \mathbb{R}$ only if $\mathcal{E}_s \ge \frac{(\beta^2+1)\rho}{(\beta^2+1)H+\beta^2 G_s}$.

Denote the admissible range $[\frac{(\beta^2+1)\rho}{(\beta^2+1)H+\beta^2 G_s}, \frac{(\beta^2+1)\rho}{\beta^2 G_s}]$ of $\mathcal{E}_s$ as $\mathcal{A}$. Given ρ and the squared channel amplitudes G_s, G_r, and H, (12) implies that $\mathcal{E}_r$ is dependent on $\mathcal{E}_s$. It can be shown that it is hard to find an explicit analytical solution to (6). Numerical solutions to (6), however, are aided by the following result.

Proposition 2. *When FRC is used at the destination, the total energy $\mathcal{E}_{\mathsf{tot}}$ is still a convex function of $\mathcal{E}_s$ on $\mathcal{A}$.*

The proof of Proposition 2 is provided in Appendix B.

Proposition 2 implies that standard numerical convex optimization methods can be used to find the unique solution to (6).

Denote $\mathcal{E}_s^*$ as the value of $\mathcal{E}_s$ that attains the minimum in (6) and note that $\mathcal{E}_r^*$ is implied by (12). Given the convexity of $\mathcal{E}_{\mathsf{tot}}$ on $\mathcal{A}$, we can determine whether the unique minimum of $\mathcal{E}_{\mathsf{tot}}$ on $\mathcal{A}$ occurs at the point $\mathcal{E}_s = \frac{(\beta^2+1)\rho}{\beta^2 G_s}$ by evaluating $\frac{\partial \mathcal{E}_{\mathsf{tot}}}{\partial \mathcal{E}_s}$ at this point. If $\frac{\partial \mathcal{E}_{\mathsf{tot}}}{\partial \mathcal{E}_s} > 0$ at $\mathcal{E}_s = \frac{(\beta^2+1)\rho}{\beta^2 G_s}$, then the minimum of $\mathcal{E}_{\mathsf{tot}}$ on $\mathcal{A}$ must occur at $\mathcal{E}_s < \frac{(\beta^2+1)\rho}{\beta^2 G_s}$ (corresponding to $\mathcal{E}_r^* > 0$), otherwise the minimum occurs at $\mathcal{E}_s = \frac{(\beta^2+1)\rho}{\beta^2 G_s}$ (corresponding to $\mathcal{E}_r^* = 0$). It can be shown that

$$\frac{\partial \mathcal{E}_{\mathsf{tot}}}{\partial \mathcal{E}_s}\bigg|_{\mathcal{E}_s = \frac{(\beta^2+1)\rho}{\beta^2 G_s}} = 1 > 0,$$

hence the unique minimum of $\mathcal{E}_{\mathsf{tot}}$ on $\mathcal{A}$ must occur at $\mathcal{E}_s < \frac{(\beta^2+1)\rho}{\beta^2 G_s}$. This implies that $\mathcal{E}_r^* > 0$ for all G_s, G_r, H, ρ. Thus in the case of FRC, the relay should always transmit, i.e. $\mathcal{E}_r^* > 0$ for all channel states. This is in contrast to the result in Section 4.1 showing that direct transmission ($\mathcal{E}_r^* = 0$) is optimum for certain channel states when the destination uses MRC.

5 Simulation Results

In this section, we present the performance of the optimum energy allocation scheme and show how the optimum total energy gain are affected by this scheme.

Proposition 1 implies when the relay does not have an advantaged channel to the destination, the total energy is minimized when all of the transmission energy is allocated to the source and the relay does not transmit. Figure 2 shows the total energy gain of optimum AF cooperative transmission when $\rho \to \infty$. Similarly, the largest gains occur when $\mu \ll 1$, which corresponds to the case where the relay has a advantaged channel to the destination and $\eta \to 1$, which corresponds to the case where the source and relay are much closer in proximity than the source and destination.

In figure 3, it can be shown that $\frac{\mathcal{E}_s^*}{\mathcal{E}_{\mathsf{tot}}^*} = 1$ when $\mu \geq 1$, i.e. the relay does not have an advantaged channel to the destination, all of the transmission energy is allocated to the source. Only when $0 \leq \mu < 1$, i.e. the relay has an advantaged channel that the total energy could be minimized through cooperation transmission. As expected, for a fixed μ, $\frac{\mathcal{E}_s^*}{\mathcal{E}_{\mathsf{tot}}^*}$ decreases when η increases, which corresponds to the case when the source-relay channel are more favorable, more transmission energy is allocated to the relay. Note that for a fixed η, when $\mu \to 0$, i.e. the relay has a much advantaged channel to the destination, the relay only needs a small amount of transmission energy to satisfy the SNR requirement, thus $\frac{\mathcal{E}_s^*}{\mathcal{E}_{\mathsf{tot}}^*}$ becomes larger.

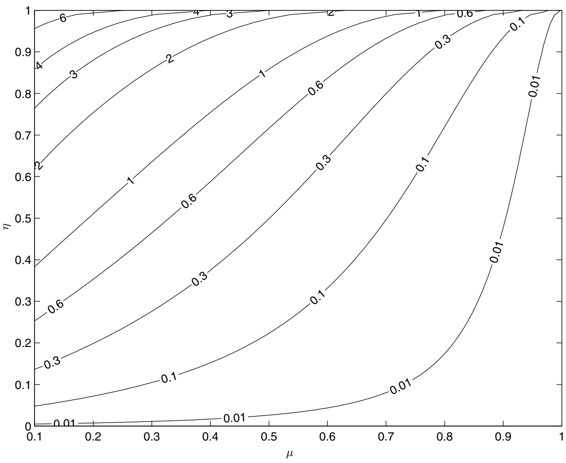

Fig. 2. $\mathcal{E}_{tot}$ gain, in dB with respect to direct transmission, of AF cooperative transmission with optimum energy allocation $\{\mathcal{E}_s^*, \mathcal{E}_r^*\}$ as a function of the parameters μ and η

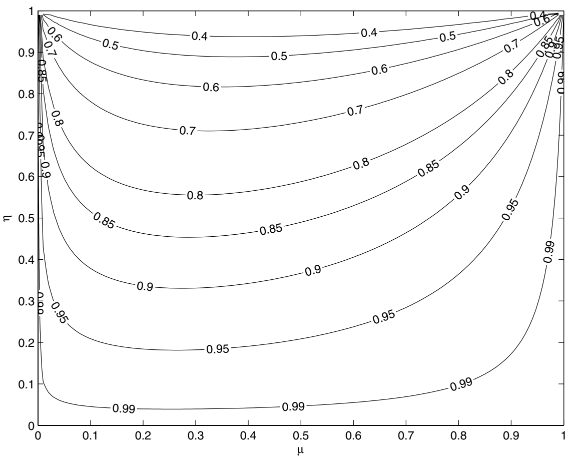

Fig. 3. Normalized optimum source energy allocation $\mathcal{E}_s^*/\mathcal{E}_{\mathsf{tot}}^*$ as a function of the parameters μ and η

6 Conclusion

This paper examines optimum energy allocation for amplify-and-forward cooperation with the goal of minimizing average total transmit energy under a SNR constraint in two scenarios: i) maximal ratio combining (MRC) and (ii) fixed ratio combining (FRC). For MRC, based on the explicit analytical solution an asymptotic solution for normalized optimum total energy in terms of μ and η

was derived in the high-SNR scenario. For FRC, we find that though it is hard to find an explicit analytical solution, standard numerical convex optimization methods can be used to find the unique solution to the problem. Based on these analysis, we explicitly describe the set of channel conditions under which the optimum energy allocation strategy can be realized.

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A Proof of Proposition 1

Proof. Before deriving the minimum weighted total energy under a minimum SNR constraint, we first determine the conditions for direct transmission and cooperative transmission. From (3), we note that the space of admissible energy allocations satisfying $\mathsf{SNR}_{\mathsf{mrc}} = \rho$ can be described as the region in $\mathbb{R}^2$ where $\mathcal{E}_r \geq 0$ and $\frac{\rho}{H+G_s} < \mathcal{E}_s \leq \frac{\rho}{G_s}$, where the upper limit to $\mathcal{E}_s$ corresponds to the case when $\mathcal{E}_r = 0$ and the lower limit corresponds to the case when $\mathcal{E}_r \to \infty$.

Using (3), the total energy required to satisfy the constraint $\mathsf{SNR}_{\mathsf{mrc}} = \rho$ can be written as

$$\mathcal{E}_{\mathsf{tot}} := \mathcal{E}_s + \alpha\mathcal{E}_r = \mathcal{E}_s + \alpha\frac{H\mathcal{E}_s^2 G_s + (G_s - H\rho)\mathcal{E}_s - \rho}{G_r(\rho - (H + G_s)\mathcal{E}_s)}. \tag{13}$$

Define the interval $\mathcal{A} = \left(\frac{\rho}{H+G_s}, \frac{\rho}{G_s}\right]$. If

$$\mathcal{E}_{\mathsf{tot}}^* = \arg\min_{\mathcal{E}_s \in \mathcal{A}} \mathcal{E}_{\mathsf{tot}} = \frac{\rho}{G_s}$$

then $\mathcal{E}_r = 0$ and $\mathcal{E}_{\mathsf{tot}}$ is minimized with direct transmission. Otherwise, $\mathcal{E}_r > 0$ and cooperative transmission minimizes $\mathcal{E}_{\mathsf{tot}}$.

In order to determine if the minimum of (13) on $\mathcal{A}$ occurs at the point $\mathcal{E}_s = \frac{\rho}{G_s}$, we first establish that (13) can have only one minimum on $\mathcal{A}$ by proving that (13) is a strictly convex function of $\mathcal{E}_s$ on $\mathcal{A}$. The second derivative of (13) with respect to $\mathcal{E}_s$ can be written as

$$\mathcal{E}_{\mathsf{tot}}'' := \frac{\partial^2}{\partial\mathcal{E}_s^2}\mathcal{E}_{\mathsf{tot}} = \alpha\frac{-2H\rho[(\rho+1)H + G_s]}{G_r(\rho - (H + G_s)\mathcal{E}_s)^3} \tag{14}$$

Note that the numerator of (14) is a negative quantity not dependent on $\mathcal{E}_s$. Since $\mathcal{E}_s(H + G_s) > \rho$ and $G_r > 0$, the denominator of (14) is also negative on the interval $\mathcal{E}_s \in \mathcal{A}$, hence $\mathcal{E}_{\mathsf{tot}}''$ is always positive on $\mathcal{A}$. This implies that $\mathcal{E}_{\mathsf{tot}}$ is a strictly convex function of $\mathcal{E}_s$ on $\mathcal{A}$.

Given the convexity of $\mathcal{E}_{\mathsf{tot}}$ on $\mathcal{A}$, we can determine whether the unique minimum of (13) on $\mathcal{A}$ occurs at the point $\mathcal{E}_s = \frac{\rho}{G_s}$ by evaluating the first derivative of (13) at this point. If the first derivative is positive, then the minimum of (13) on $\mathcal{A}$ must occur at $\mathcal{E}_s < \frac{\rho}{G_s}$ (corresponding to cooperative transmission), otherwise the minimum occurs at $\mathcal{E}_s = \frac{\rho}{G_s}$ (corresponding to direct transmission). The first derivative of (13) evaluated at $\mathcal{E}_s = \frac{\rho}{G_s}$ can be written as

$$\mathcal{E}_{\mathsf{tot}}'\left(\frac{\rho}{G_s}\right) := \frac{\partial}{\partial\mathcal{E}_s}\mathcal{E}_{\mathsf{tot}}\left(\frac{\rho}{G_s}\right) = 1 - \alpha\frac{G_s(H\rho + G_s)}{G_r H\rho} \tag{15}$$

This quantity is positive if and only if the condition of $\frac{\alpha G_s}{G_r}\left(1 + \frac{G_s}{H\rho}\right) < 1$ are satisfied, i.e. $\mu < 1$, hence the unique minimum of (13) on $\mathcal{A}$ must occur at $\mathcal{E}_s < \frac{\rho}{G_s}$ when $0 \le \mu < 1$. Otherwise, when $\mu \ge 1$ the minimum of (13) on $\mathcal{A}$ must occur at $\mathcal{E}_s = \frac{\rho}{G_s}$ and direct transmission is optimum.

We now derive the explicit solution to the total energy minimization problem. The optimal source energy allocation can be found by solving $\frac{\partial}{\partial\mathcal{E}_s}\mathcal{E}_{\mathsf{tot}} = 0$. Computation of the partial derivative and algebraic simplification yields

$$1 + \frac{\alpha(H(2G_s\mathcal{E}_s - \rho) + G_s)}{G_r(\rho - (H + G_s)\mathcal{E}_s)} + \frac{\alpha(G_s\mathcal{E}_s(1 + H) - \rho(1 + H\mathcal{E}_s))}{G_r(\rho - (H + G_s)\mathcal{E}_s)^2} = 0 \tag{16}$$

By solving this equation for $\mathcal{E}_s$, the correct root which satisfies $\frac{\partial^2}{\partial\mathcal{E}_s^2}\mathcal{E}_{\mathsf{tot}}(\mathcal{E}_s) \ge 0$ is

$$\mathcal{E}_s = \frac{\rho}{H + G_s} + \frac{\sqrt{\alpha\rho H(G_s + (1+\rho)H)}}{(H + G_s)\sqrt{H(G_r - \alpha G_s) + G_s G_r}} \tag{17}$$

In the case $0 \leq \mu < 1$, the total energy can be minimized through cooperative transmission. By plugging $\mathcal{E}_s^*$ into (14), the minimized total energy can be expressed as

$$\mathcal{E}_{\mathsf{tot}}^* = \frac{\left(\sqrt{\rho(G_r(G_s+H)-\alpha G_s H)}+\sqrt{\alpha H(G_s+H+\rho H)}\right)^2-\alpha(G_s+H)^2}{G_r(G_s+H)^2}$$

B Proof of Proposition 2

Proof. To prove $\mathcal{E}_{\mathsf{tot}}$ is convex, and hence has a unique minimum on $\mathcal{A} = [\frac{(\beta^2+1)\rho}{(\beta^2+1)H+\beta^2 G_s}, \frac{(\beta^2+1)\rho}{\beta^2 G_s}]$, we will show that $\frac{\partial^2 \mathcal{E}_{\mathsf{tot}}}{\partial \mathcal{E}_s^2} > 0$ a.s. Here, $\frac{\partial^2 \mathcal{E}_{\mathsf{tot}}}{\partial \mathcal{E}_s^2}$ is a function of $\mathcal{E}_s$. Substitute $\mathcal{E}_s$ with y, we have

$$\frac{\partial^2 \mathcal{E}_{\mathsf{tot}}}{\partial \mathcal{E}_s^2} = \mathsf{F}(y)\mathsf{G}(y). \tag{18}$$

The function

$$\begin{aligned}\mathsf{F}(y) = {} & \beta y^4 + 4\beta^2 G_s \rho y^3 - (6(\beta^3+\beta)\rho^2 G_s H + 3\beta^3 \rho G_s^2 + 3(\beta^3+\beta)\rho G_s H) y^2 \\ & + (4(\beta^2+1)^2 \rho^3 H^2 G_s + 4(\beta^4+\beta^2)\rho^2 H G_s^2 + 4(\beta^2+1)^2 \rho^2 H^2 G_s) y \\ & + (\beta^5+2\beta^3+\beta) G_s^2 H^2 \rho^4 + (\beta^5+\beta^3) G_s^3 H \rho^3 + (\beta^5+2\beta^3+\beta) H^2 G_s^2 \rho^3.\end{aligned} \tag{19}$$

and

$$\mathsf{G}(y) = \frac{\alpha\rho^2 G_s^2 H(\beta^2 G_s + \beta^2 H + H)^2}{2G_r(y+\beta\rho G_s)^4 y^3} \tag{20}$$

where $y := \sqrt{G_s H \rho((\beta^2+1)H\mathcal{E}_s + \beta^2 G_s \mathcal{E}_s - (\beta^2+1)\rho)}$.

Note that the squared channel amplitudes G_s, G_r and H are exponentially distributed, thus $\lim_{\epsilon\to 0} P\{X \leq 0\} = 0$, where X denotes the squared channel amplitudes. Thus $\mathsf{G}(y) > 0$ a.s. on $\mathcal{A}$ (Note that $y \neq 0$). Hence, the condition $\frac{\partial^2 \mathcal{E}_{\mathsf{tot}}}{\partial \mathcal{E}_s^2} > 0$ a.s. on $\mathcal{A} \Leftrightarrow \mathsf{F}(\mathsf{y}) > 0$ a.s. on $\mathcal{C}$, where $\mathcal{C} = \left[0, \frac{(\beta^2+1)\rho H}{\beta}\right]$. Observe that only the y^2 term has negative coefficient. The function $\mathsf{F}(y)$ can be written as

$$\mathsf{F}(y) = \mathsf{R}(y) + \mathsf{S}(y) + \mathsf{T}(y), \tag{21}$$

where

$$\begin{aligned}\mathsf{R}(y) &= -(3\beta^3\rho G_s^2 + 3(\beta^3+\beta)\rho G_s H) y^2 + (4(\beta^4+\beta^2)\rho^2 H G_s^2 + 4(\beta^2+1)^2 \rho^2 H^2 G_s) y \\ \mathsf{S}(y) &= 4\beta^2 G_s \rho y^3 - 6(\beta^3+\beta)\rho^2 G_s H y^2 + 4(\beta^2+1)^2 \rho^3 H^2 G_s y \\ \mathsf{T}(y) &= \beta y^4 + (\beta^5+2\beta^3+\beta) G_s^2 H^2 \rho^4 + (\beta^5+\beta^3) G_s^3 H \rho^3 + (\beta^5+2\beta^3+\beta) H^2 G_s^2 \rho^3.\end{aligned}$$

Note that $\mathsf{T}(y) \geq 0$ for $\left[0, \frac{(\beta^2+1)\rho H}{\beta}\right]$. We will consider the behavior of $R(y)$ and $S(y)$ in following claims.

Claim 1: $\mathsf{R}(y) > 0$ a.s. on $\mathcal{C}$.
proof: Observe that $\mathsf{R}(y)$ is a quadratic equation of one variable. It can be written as

$$\mathsf{R}(y) = y\mathsf{r}(y), \tag{22}$$

where

$$\mathsf{r}(y) = -(3\beta^3\rho G_s^2 + 3(\beta^3+\beta)\rho G_s H)y + 4(\beta^4+\beta^2)\rho^2 H G_s^2 + 4(\beta^2+1)^2\rho^2 H^2 G_s.$$

First, we consider the case when $\beta > 0$. Observe that $y > 0$ and $\mathsf{r}(\frac{(\beta^2+1)\rho \mathsf{H}}{\beta}) = (\beta^4+\beta^2)\rho^2 H G_s^2 + (\beta^2+1)^2\rho^2 H^2 G_s > 0$ a.s. Thus, to prove $\mathsf{R}(y) > 0$ a.s. on $\mathcal{C}$, it is only necessary to prove that $\mathsf{r}(y)$ is decreasing on $\mathcal{C}$. It can be shown that

$$\frac{\partial \mathsf{r}(y)}{\partial y} = -3\beta^3\rho G_s^2 - 3(\beta^3+\beta)\rho G_s H < 0 \;\; a.s. \tag{23}$$

Thus, $\mathsf{r}(y) > 0$ a.s. on $\mathcal{C}$, this result implies $\mathsf{R}(y) > 0$ a.s. on $\mathcal{C}$. When $\beta = 0$, $\mathsf{R}(y) = 4y\rho^2 H^2 G_s > 0$ a.s. on $\mathcal{C}$.

Claim 2: $\mathsf{S}(y) > 0$ a.s. on $\mathcal{C}$.
proof: Observe that $\mathsf{S}(y)$ is a cubic equation of one variable. It can be written as

$$\mathsf{S}(y) = y\mathsf{s}(y), \tag{24}$$

where

$$\mathsf{s}(y) = 4\beta^2 G_s \rho y^2 - 6(\beta^3+\beta)\rho^2 G_s H y + 4(\beta^2+1)^2\rho^3 H^2 G_s.$$

First, we consider the case when $\beta > 0$. Observe that $y > 0$ and $\mathsf{s}(y)$ is a quadratic equation. To prove that $\mathsf{S}(y) > 0$ a.s. on $\mathcal{C}$, it is only necessary to prove that $\mathsf{s}(y) > 0$ a.s. on $\mathcal{C}$. It can be shown that $\frac{\partial^2 \mathsf{s}(y)}{\partial y^2} = 4\beta^2 G_s \rho > 0$ a.s., hence $\mathsf{s}(y)$ is convex on $\mathcal{C}$. Thus, to prove $\mathsf{s}(y) > 0$ a.s. on $\mathcal{C}$, it is only necessary to prove that $\mathsf{s}(y)$ has no real root a.s. It can be shown

$$\begin{aligned}[6(\beta^3+\beta)\rho^2 G_s H]^2 - 4(4\beta^2 G_s\rho)[4(\beta^2+1)^2\rho^3 H^2 G_s] = \\ -28\beta^2(\beta^2+1)^2\rho^4 G_s^2 H^2 < 0 \;\; a.s.\end{aligned}$$

This implies that $\mathsf{s}(y) > 0$ a.s. on $\mathcal{C}$, which implies $\mathsf{S}(y) > 0$ a.s. on $\mathcal{C}$. When $\beta = 0$, $\mathsf{S}(y) = 4y\rho^3 H^2 G_s > 0$ a.s. on $\mathcal{C}$.