Multi-user Broadcast Authentication in Wireless Sensor Networks

Kui Ren  
Worcester Polytechnic Institute,  
Worcester, MA 01609  
kren@ece.wpi.edu

Wenjing Lou  
Worcester Polytechnic Institute,  
Worcester, MA 01609  
wjlou@ece.wpi.edu

Yanchao Zhang  
New Jersey Institute of Technology,  
Newark, NJ 07102  
yczhang@njit.edu

Abstract—Broadcast authentication is a critical security service in wireless sensor networks (WSNs), as it allows the mobile users of WSNs to broadcast messages to multiple sensor nodes in a secure way. Although symmetric-key-based solutions such as $\mu$TESLA and multilevel $\mu$TESLA have been proposed, they all suffer from severe energy-depletion attacks resulted from the nature of delayed message authentication. This paper presents several efficient public-key-based schemes to achieve immediate broadcast authentication and thus avoid the security vulnerability intrinsic to $\mu$TESLA-like schemes. Our schemes are built upon the unique integration of several cryptographic techniques, including the Bloom filter, the partial message recovery signature scheme and the Merkle hash tree. We prove the effectiveness and efficiency of the proposed schemes by a comprehensive quantitative analysis of their energy consumption in both computation and communication.

I. INTRODUCTION

Wireless sensor networks (WSNs) have enabled data gathering from a vast geographical region and present unprecedented opportunities for a wide range of tracking and monitoring applications from both civilian and military domains [1], [2]. In these applications, WSNs are expected to process, store, and provide the sensed data to the network users upon their demands [20]. As the most common communication paradigm, the network users are expected to issue the queries to the network in order to obtain the information of their interest. Furthermore, in wireless sensor and actuator networks (WSANs) [2], the network users may need to issue their commands to the network (probably based on the information they received from the network). In both cases, there could be a large number of users in the WSNs, which may be either mobile or static; and the users may use their mobile clients to query or command the sensor nodes from anywhere in the WSN. Obviously, broadcast/multicast operations are fundamental to the realization of these network functions. Hence, it is also highly important to ensure broadcast authentication for the security purpose.

Broadcast authentication in WSNs was first addressed by $\mu$TESLA [26]. In $\mu$TESLA, users of WSNs are assumed to be one or a few fixed sinks, which are always assumed to be trustworthy. The scheme adopts a one-way hash function $h(\cdot)$ and uses the hash preimages as keys in a message authentication code (MAC) algorithm. Initially, sensor nodes are preloaded with $K_0 = h^a(x)$, where $x$ is the secret held by the sink. Then, $K_1 = h^{a-1}(x)$ is used to generate MACs for all the broadcast messages sent within time interval $I_1$. During time interval $I_2$, the sink broadcasts $K_1$, and sensor nodes verify $h(K_1) = K_0$. The authenticity of messages received during time interval $I_1$ are then verified using $K_1$. This delayed disclosure technique is used for the entire hash chain and thus demands loosely synchronized clocks between the sink and sensor nodes. $\mu$TESLA is later enhanced in [17] to overcome the length limit of the hash chain. Most recently, $\mu$TESLA is also extended in [18] to support multiuser scenario but the scheme assumes that each sensor node only interacts with a very limited number of users.

It is generally held that $\mu$TESLA-like schemes have the following shortcomings even in the single-user scenario: 1) all the receivers have to buffer all the messages received within one time interval; 2) they are subject to Wormhole attacks [11], where messages could be forged due to the propagation delay of the disclosed keys. However, here we point out a much more serious vulnerability of $\mu$TESLA-like schemes when they are applied in multi-hop WSNs. Since sensor nodes buffer...
all the messages received within one time interval, an adversary can hence flood the whole network arbitrarily. All he has to do is to claim that the flooding messages belong to the current time interval which should be buffered for authentication until the next time interval. Since wireless transmission is very expensive in WSNs, and WSNs are extremely energy constrained, the ability to flood the network arbitrarily could cause devastating Denial of Service (DoS) attacks. Moreover, this type of energy-depletion DoS attacks become more devastating in multiuser scenario as the adversary now can have more targets and hence more chances to generate bogus messages without being detected. Obviously, all these attacks are due to delayed authentication of the broadcast messages. In [11], TIK is proposed to achieve immediate key disclosure and hence immediate message authentication based on precise time synchronization between the sink and receiving nodes. However, this technique is not applicable in WSNs as pointed out by the authors. Therefore, multiuser broadcast authentication still remains a wide open problem in WSNs.

When $\mu$TESLA was proposed, sensor nodes were assumed to be extremely resource-constrained, especially with respect to computation capability, bandwidth availability, and energy supply [26]. Therefore, public key cryptography (PKC) was thought to be too computationally expensive for WSNs, though it could provide much simpler solutions with much stronger security resilience. At the same time, the computationally efficient one-time signature schemes are also considered unsuitable for WSNs, as they usually involve intense communications [26]. However, recent studies [8], [28], [31] showed that, contrary to widely held beliefs, PKC with even software implementations only is very viable on sensor nodes. For example [31], Elliptic Curve Cryptography (ECC) signature verification takes 1.61s with 160-bit keys on ATMega128 8MHz, a processor used in current Crossbow motes platform [7]. Furthermore, the computational cost is expected to fall faster than the cost to transmit and receive. For example, ultra-low-power microcontrollers such as the 16-bit Texas Instruments MSP430 [30] can execute the same number of instructions at less than half the power required by the 8-bit ATMega128L. The benefits of transmitting shorter ECC keys and hence shorter messages/signatures will in turn be more significant. Moreover, next generation sensor nodes are expected to combine ultra-low power circuitry with so-called power scavengers such as Heliomote [15], which allow continuous energy supply to the nodes. At least $8 - 20\mu W$ of power can be generated using MEMS-based power scavengers [3]. Other solar-based systems are even able to deliver power up to 100mW for the MICA Motes [15], [16]. These results indicate that, with the advance of fast growing technology, PKC is no longer impractical for WSNs, though still expensive for the current generation sensor nodes, and its wide acceptance is expected in the near future [8].

Having this observation and knowing that symmetric-key-based solutions such as $\mu$TESLA are insufficient for broadcast authentication in WSNs, we resort to PKC for more effective solutions. In this paper, we address multiuser broadcast authentication problem in WSNs by designing PKC-based solutions with minimized computational and communication costs.

Overview of the paper: In this paper, we propose four different public-key-based approaches and provide in-depth analysis of their advantages and disadvantages. In all the four approaches, the users are always authenticated through their public keys. We first propose a straightforward certificate-based approach and point out its high energy inefficiency with respect to both communication and computation costs. We then propose a direct storage based scheme, which has high efficiency but suffers from the scalability problem. A Bloom filter based scheme is further proposed to improve the memory efficiency over the direct storage based scheme. Further techniques are also developed to increase the security strength of the proposed scheme. Lastly, we propose a hybrid scheme to support a larger number of network users by employing the Merkle hash tree technique. We give an in-depth quantitative analysis of the proposed schemes and demonstrate their effectiveness and efficiency in WSNs in terms of energy consumption.

Contributions: This paper makes the following contributions: 1) We identify the problem of multiuser broadcast authentication in WSNs and point out a serious security vulnerability inherent to the symmetric-key based $\mu$TESLA-like schemes. 2) We come up with several PKC-based schemes to address the proposed problem with minimized computational and communication costs. We achieve our goal by integrating several cryptographic building blocks, such as the Bloom filter, the partial message recovery signature scheme, and the Merkle hash tree, in an innovative manner. 3) We analyze both the performance and security resilience of the proposed schemes. A quantitative energy consumption analysis is given in detail and demonstrates the effectiveness and efficiency of the proposed schemes.

Organization of the paper: The remaining part of this paper is as follows: In Section II, we introduce
the cryptographic mechanisms to be used. Section III presents the system assumption, adversary model, and security objectives. In Section IV, we introduce two basic schemes. We further propose two advanced schemes and detail the underlying design logic in Section V. Section VI analyzes the performance of the proposed schemes, and we conclude our paper in Section VII.

II. PRELIMINARIES

The Bloom Filter: A Bloom filter is a simple space-efficient randomized data structure for representing a set in order to support membership queries [23]. A Bloom filter for representing a set \( S = s_1, s_2, ..., s_n \) of \( n \) elements is described by a vector \( \mathcal{V} \) of \( m \) bits, initially all set to 0. A Bloom filter uses \( k \) independent hash functions \( h_1, ..., h_k \) with range \( 0, 1, ..., m-1 \), which map each item in the universe to a random number uniform over \( [0, ..., m-1] \). For each element \( s \in S \), the bits \( h_i(s) \) are set to 1 for \( 1 \leq i \leq k \). Note that a bit of \( \mathcal{V} \) can be set to 1 multiple times. To check if an item \( x \) is in \( S \), we check whether all bits \( h_i(x) \) are set to 1. If not, \( x \) is not a member of \( S \) for certain, that is, no false negative error. If yes, \( x \) is assumed to be in \( S \). A Bloom filter may yield a false positive. It may suggest that an element \( x \) is in \( S \) even though it is not. The probability of a false positive for an element not in the set can be calculated as follows. After all the elements of \( S \) are hashed into the Bloom filter, the probability that a specific bit is still 0 is \((1 - \frac{1}{m})^{kn} \approx e^{-kn/m} \). The probability of a false positive \( f \) is then \( f = (1 - (1 - \frac{1}{m})^{kn})^k \approx (1 - e^{-kn/m})^k \).

The Merkle Hash Tree: A Merkle Tree is a construction introduced by Merkle in 1979 to build secure authentication schemes from hash functions [22]. It is a tree of hashes where the leaves in the tree are hashes of the authentic data values \( n_1, n_2, ..., n_w \). Nodes further up in the tree are the hashes of their respective children. For instance, assuming that \( w = 4 \) in Fig. 1, the values of the four leaf nodes are the hashes of the data values, \( h(n_i), i = 1, 2, 3, 4 \), respectively, under a one-way hash function \( h() \) (e.g., SHA-1 [25]). The value of an internal node \( A \) is \( h_A = h(h(n_1))\|h(n_2)) \), and the value of the root node is \( h_r = h(h_a\|h_b) \). \( h_r \) is used to commit to the entire tree to authenticate any subset of the data values \( n_1, n_2, n_3, \) and \( n_4 \) in conjunction with a small amount of auxiliary authentication information \( \text{AAI} \) (i.e., \( \log_2 N \) hash values where \( N \) is the number of leaf nodes). For example, a receiver with the authentic \( h_r \) requests for \( n_3 \) and requires the authentication of the received \( n_3 \). The source sends the \( \text{AAI} : < h_a, h(n_4) > \) to the receiver. The receiver can then verify \( n_3 \) by first computing \( h(n_3) \),

\[
h_b = h(h(n_3))\|h(n_4))\) and \( h_r = h(h_a\|h_b)).
\]

and then checking if the calculated \( h_r \) is the same as the authentic root value \( h_r \).

III. SYSTEM MODEL, ADVERSARY MODEL, AND DESIGN GOALS

System Model: In this paper, we consider a large spatially distributed WSN, consisting of a fixed sink(s) and a large number of sensor nodes. The sensor nodes are usually resource-constrained with respect to memory space, computation capability, bandwidth, and power supply. The WSN is aimed to offer information services to many network users that roam in the network, in addition to the fixed sink(s) [20]. The network users may include mobile sinks, vehicles, and people with mobile clients, and they are assumed to be more powerful than sensor nodes in terms of computation and communication abilities. For example, the network users could consist of a number of doctors, nurses, medical equipment (acting as actuators) and so on, in the case of CodeBlue [19], where the WSN is used for emergency medical response. These network users broadcast queries.commands through sensor nodes in the vicinity, and expect the replies that reflect the latest network information. The network users can also communicate with the sink or the backend server directly without going through the WSN if necessary. We assume that the sink is always trustworthy but the sensor nodes are subject to compromise. At the same time, the users of the WSN may be dynamically revoked due to either membership changes or compromise, and the revocation pattern is not restricted. We also assume that the WSN is loosely synchronized.

Adversary Model: In this paper, we assume that the adversary’s goal is to inject bogus messages into the network, attempt to deceive sensor nodes, and obtain the information of his interest. Additionally, Denial of Service (DoS) attacks such as bogus message flooding, aiming at exhausting constrained network resources, is another important focus of the paper. We assume that
the adversary is able to compromise both network users and the sensor nodes. The adversary hence could exploit the compromised users/nodes for such attacks. However, we do assume that adversary cannot compromise an unlimited number of sensor nodes.

**Design Goals:** Our security goal is straightforward: all messages broadcasted by the network users of the WSN should be authenticated so that the bogus ones inserted by the illegitimate users and/or compromised sensor nodes can be efficiently rejected/filtered. We also focus on minimizing the overheads of the security design. Especially, energy efficiency (with respect to both communication and computation) and storage overhead are given priority to cope with the resource-constrained nature of WSNs.

**IV. THE BASIC SCHEMES**

**A. The Certificate-Based Authentication Scheme (CAS)**

CAS works as follows. Each user (not a sensor) of the WSN is equipped with a public/private key pair (PK/SK), and signs every message he broadcasts with his SK using a digital signature scheme such as ECDSA [10]. Note that in all our designs, we do not require sensors to have public/private key pairs for themselves. To prove the user’s ownership over his public key, the sink is also equipped with a public/private key pair and serves as the certification authority (CA). The sink issues each user a public key certificate, which, to its simplest form, consists of the following contents: $\text{Cert}_{U_D} = U_D, \text{PK}_{U_D}, \text{Exp}_T, \text{SIG}_{\text{SK}_{\text{Sink}}} \{h(U_D||[\text{Exp}_T||\text{PK}_{U_D}])\}$, where $U_D$ denotes the user’s ID, $\text{PK}_{U_D}$ denotes its public key, $\text{Exp}_T$ denotes certificate expiration time, and $\text{SIG}_{\text{SK}_{\text{Sink}}} \{h(U_D||[\text{Exp}_T||\text{PK}_{U_D}])\}$ is a signature over $h(U_D||[\text{Exp}_T||\text{PK}_{U_D}])$ with $\text{SK}_{\text{Sink}}$. Hence, a broadcast message is now of the form as follows:

\[
< M, tt, \text{SIG}_{\text{SK}_{U_D}} \{h(U_D||tt||M)\}, \text{Cert}_{U_D} > \quad (I)
\]

Here, $M$ denotes the broadcast message and $tt$ denotes the current time. For the purpose of message authentication, sensor nodes are preloaded with $\text{PK}_{\text{Sink}}$ before the network deployment; and message verification contains two steps: the user certificate verification and the message signature verification.

CAS suffers from two main drawbacks. First and foremost, it is not efficient in communication, as the certificate has to be transmitted along with the message across every hop as the message propagates in the WSN. A large per message overhead will result in more energy consumption on every single sensor node. In CAS, the per message overhead is as high as $|tt| + |\text{SIG}_{\text{SK}_{U_D}} \{h(U_D||M)\}| + |\text{Cert}_{U_D}| = 128$ bytes. As in [31], the user certificate is at least 86 bytes, when ECDSA-160 [10] is used. Here, we assume that $tt$ and $U_D$ are both two bytes, in which case the scheme supports up to 65,535 network users. Moreover, $|\text{SIG}_{\text{SK}_{U_D}} \{h(U_D||M)\}| = 40$ bytes, when ECDSA-160 [10] is assumed. Second, to authenticate each message, it always takes two expensive signature verification operations. This is because the certificate should always be authenticated in the first place.

**B. The Direct Storage Based Authentication Scheme (DAS)**

One way to reduce the per message overhead and the computational cost is to eliminate the existence of the certificate. A straightforward approach is then to let sensor nodes simply store all the current users’ ID information and their corresponding public keys. In this way, a broadcast message now only contains the following contents:

\[
< M, tt, \text{SIG}_{\text{SK}_{U_D}} \{h(U_D||tt||M)\}, U_D, \text{PK}_{U_D} > . \quad (II)
\]

Verifying the authenticity of a user public key is reduced to finding out whether or not the attached user/public key pair is contained in the local memory. Upon user revocation, the sink simply sends out ID information of the revoked user, and every sensor node deletes the corresponding user/public key pair in its memory.

The drawbacks of DAS are obvious. Given a storage limit of 5 KB, only 232 users can be supported at most; even with a memory space of 19.5 KB, DAS can only support up to 1,000 users. At the same time, CAS can support up to 2,560 users given the same storage limit 5 KB. The reason is that in CAS only the ID information of the revoked users are stored by the sensor nodes. Therefore, DAS is neither memory efficient nor scalable.

However, the advantage of DAS is also significant as compared to CAS. It successfully reduces the per message overhead down to $|tt| + |\text{SIG}_{\text{SK}_{U_D}} \{h(U_D||M)\}| + |U_D| + |\text{PK}_{U_D}| = 64$ bytes. The above analysis clearly shows that more advanced schemes are needed other than DAS and CAS. And the direction to seek is to improve storage efficiency while retaining or further reducing the per message overhead.

**V. THE ADVANCED SCHEMES**

**A. The Bloom Filter Based Authentication Scheme (BAS)**

We assume that the sink represents the network planner.
System Preparation: The sink generates the public keys for all network users, and constructs the set:

$$ S = \{<U_{ID_1}, PK_{U_{ID_1}}>, <U_{ID_2}, PK_{U_{ID_2}}>, \ldots \}, $$

where \(|S| = N\), and \(|\cdot\|\) denotes the cardinality of the set. Using the Bloom filter, the sink can apply \(k\) system-wide hash functions (cf. Section II.B) to map the elements of \(S\) (each with \(L + 2\) bytes, that is, \(|U_{ID}| = 2\) bytes, and \(|PK_{U_{ID}}| = L \) bytes) to an \(m\)-bit vector \(V\) with \(V = v_0v_1\ldots v_{m-1}\), where we have \(m < N(L + 2)\) to reduce the filter size and \(m > kN\) to retain a small probability of a false positive. These \(k\) hash functions are known by every node and the sink. For each \(v_i, i \in [0, m - 1]\), we have

$$ v_i = \begin{cases} 1, & \text{if } \exists l \in [1, k], j \in [1, N], \\ s.t. h_j(U_{ID_1} \| PK_{U_{ID_j}}) = i \\ 0, & \text{otherwise} \end{cases} $$

Additionally, the sink constructs a counting Bloom filter \(\overline{V}\) of \(m + c\) bits with \(\overline{V} = \overline{v}_0\overline{v}_1\ldots\overline{v}_{m-1}\), where each \(\overline{v}_i, i \in [0, m - 1]\) is a \(c\)-bit counter, i.e., \(\overline{v}_i = c\) bits. The value of \(\overline{v}_i\) is determined as follows:

$$ \overline{v}_i = \#\{(ID_j, PK_{U_{ID_j}}) | h_j(U_{ID_1} \| PK_{U_{ID_j}}) = i, \text{ for } \exists l \in [1, k], j \in [1, N]\}. $$

And \(c = \lceil \log_2(\max(\overline{v}_i, i \in [0, m - 1])) \rceil\) bits, which is usually of 4 bits for most applications [9]. The above operations are illustrated in Fig. 2. The sink finally preloads each sensor node with \(V\) (not including \(\overline{V}\)), as well as the sink’s public key and the common parameters of the ECDSA signature scheme.

Message Signing and Authentication: Let \(PK_{U_{ID}} = W_{pub} = sG\), be the public key of user \(U_{ID}\), where \(s\) is the private key of the signer, and \(G\) is the generator of a subgroup of an elliptic curve group of order \(r\). Let \(S_K(\cdot)\) be a symmetric key cipher such as AES. To broadcast a message \(M\) (\(|M| \geq 10\) bytes), \(U_{ID}\) takes the steps below following [24], a variant of ECDSA with the partial message recovery property:

- Concatenate \(<M|tt||U_{ID}>\), and break it into two parts, \(M_1\) and \(M_2\), where \(|M_1| \leq 10\) bytes.
- Generate a random key pair \(\{u, V\}\), where \(u \in [1, r - 1]\), \(V = uG = (x_1, y_1)\), and \((x_1 \mod r) \neq 0\).
- Encode-and-hash \(V\) into an integer \(I\) [24].
- Form \(F_1\) from \(M_1\) by adding the proper redundancy [12].
- Compute \(C = (I + F_1) \mod r\), and make sure that \(C \neq 0\) or repeat the above steps otherwise.

- Compute \(F_2 = h(M_2)\), and \(D = u^{-1}(F_2 + sC)\mod r\).
- Repeat all the above steps if \(D = 0\); Output the signature as \(<C, D>\) otherwise.

Then, \(U_{ID}\) broadcasts

$$ <M_2, C, D, W_{pub}>. \quad (III) $$

where \(tt\) and \(U_{ID}\) are parts of \(M_2\). And this is the known simplest message format that can be achieved using \(PKC\). Now, upon receiving a broadcast message (not from the sink), a sensor node checks the authenticity of the message in two steps. First, it checks the authenticity of the corresponding public key by verifying its membership in \(S\). To do so, the sensor node checks whether \(v[lh(U_{ID} \| PK_{U_{ID}})]^2 = 1, l \in [1, k]\), and a negative result will lead to the discarding of the message. We note that here a false positive may happen due to the probabilistic nature of the Bloom filter, but only with a very small (negligible) probability when appropriate parameters are chosen as we will analyze later. Second, it verifies the attached signature as follows:

- Discard the message if \(C \notin [1, r-1]\) or \(D \notin [1, r-1]\).
- Compute \(F_2 = h(M_2)\), \(H = D^{-1} \mod r\), and \(H_1 = F_2H \mod r\).
- Compute \(H_2 = CH \mod r\), and \(P = H_1G + H_2W_{pub}\).
- Discard the message if \(P = 0\).

\(The\ claim\ is\ true\ only\ when\ ID-based\ cryptography\ [29]\ is\ excluded\ from\ consideration,\ in\ which\ case\ the\ user’s\ ID\ is\ also\ his\ public\ key.\ Furthermore,\ the\ shortest\ signature\ size\ possibly\ obtained\ from\ pairing\ is\ around\ 22\ bytes\ [6],\ which\ is\ shorter\ than\ 40\ bytes\ obtained\ from\ ECDSA.\ However,\ to\ apply\ a\ pairing-based\ scheme\ (i.e.,\ a\ ID-based\ signature\ or\ short\ signature)\ on\ sensor\ nodes,\ the\ known\ reachable\ signature\ size\ has\ to\ be\ 84\ bytes,\ even\ when\ a\ 32-bit\ microprocessor\ can\ be\ used\ [32].\ And\ the\ energy\ cost\ is\ also\ multiple\ times\ higher\ than\ that\ of\ an\ ECDSA-160\ signature.\)
- Encode-and-hash $P$ into an integer $I$ [24] and compute $F_1 = c - I \mod r$.
- Discard the message if the redundancy of $F_1$ is incorrect.
- Otherwise accept $M_1$ (obtained from $F_1$) and the signature and reconstruct $M||t||U_{ID} = M_1|M_2$.

**User Revocation/Addition:** To revoke a user, say $U_{ID}$, the sink follows the steps below:

- First, it hashes $h_i(U_{ID}||PK_{U_{ID}}) = i$ and decreases $\overline{v}_i$ by 1. It repeats this operation for all $h_i, i \in [1, k]$.
- From the updated counting Bloom filter $\mathcal{V}$, the sink obtains the corresponding updated Bloom filter $\mathcal{V}'$ with $\mathcal{V}' = v'_1\cdots v'_{m-1}$. Here, $v'_i = 1$ only when $v_i \geq 1$, and $v'_i = 0$ otherwise.
- The sink further calculates $\mathcal{V}_\Delta = \mathcal{V}' \oplus \mathcal{V}$ and deletes $\mathcal{V}$ afterwards. Here $\oplus$ denotes bitwise exclusive OR operation. Obviously, $\mathcal{V}_\Delta$ can be simply represented by enumerating its 1-valued bits, requiring $\overline{k}\log_2 m$ bits for indexing ($\overline{k} \leq k$). This representation is efficient for a small $\overline{k}$ as will be analyzed in Section VI.B.
- The sink finally broadcasts $\mathcal{V}_\Delta$ after signing it. The message format follows (III) but with the sink’s public key omitted, as every sensor already has it.
- Upon receiving and successfully authenticating the broadcast message, every sensor node updates its own Bloom filter accordingly, that is, if $v_{\Delta,i} = 1$, then $v_i = 0, i \in [0, m - 1]$.

BAS also supports simultaneous multiuser revocation. Suppose that $N_{rev}$ users are revoked simultaneously. The sink follows the same manner to construct $\mathcal{V}_\Delta$ with $\overline{k}$ bits set 1. Now we have $\overline{k} \leq kN_{rev}$. Furthermore, the compressed message for representing $\mathcal{V}_\Delta$ now could achieve $\overline{m}H(p)$ bits theoretically, where $H(p) = -p \log_2 p - (1 - p) \log_2 (1 - p)$ is the entropy function and $p = (1 - \frac{1}{m})^k$ is the probability of each bit being 0 in $\mathcal{V}_\Delta$. As pointed out in [23], using arithmetic coding technique can efficiently approach this lower bound.

BAS supports dynamic user addition in two ways. First, it enables a later binding of network users and their (ID, public key) pairs. In this approach, the sink may generate more (ID, public key) pairs than needed during system preparation. When a new network user joins the WSN, it will be assigned an unused ID and public key pair by the sink. Second, BAS could add new network users after the revocation of old members. This approach, however, could only add the same number of new users as that of the revoked. This requirement ensures that the probability of a false positive never increases in BAS. To do so, the sink updates its counting Bloom filter by hashing the new user’s information into the current Bloom filter. The sink then obtains a $\mathcal{V}_\Delta$ in the same way as in the revocation case, and broadcasts it after compression. This time, if $v_{\Delta,i} = 1$, sensor nodes will set $v_i = 1, i \in [0, m - 1]$ to update their current Bloom filters.

**B. Minimize the Probability of a False Positive**

Since the Bloom filter provides probabilistic membership verification only, it is important to make sure that the probability of a false positive is as small as possible.

**Theorem 1:** Given the number of network users $N$ and the storage space $m$ bits for a single Bloom filter, the minimum probability of a false positive $f$ that can be achieved is $2^{-k}$ with $k = \frac{N}{m} \ln 2$, that is,

$$f = (0.6185)^{\frac{N}{m}}.$$

**Proof:** since $f = (1 - (1 - \frac{1}{m})^k)^k \approx (1 - e^{-kN/m})^k$, we have $f = e^{k\ln(1-e^{-N/m})}$. Let $g = k\ln(1-e^{-N/m})$. Hence, minimizing $f$ is equivalent to minimizing $g$ with respect to $k$. We find

$$\frac{dg}{dk} = \ln(1 - e^{-N/m}) + \frac{kN}{m} e^{-N/m} \left(1 - e^{-N/m}\right).$$

It is easy to check that the derivative is 0 when $k = \frac{N}{m} \ln 2$. And it is not hard to show that this is a global minimum [23]. Note that in practice, $k$ must be an integer. □

Fig. 3 shows the probability of a false positive $f$ as a function of $\frac{N}{m}$, i.e., bits per element. We see that $f$ decreases sharply as $\frac{N}{m}$ increases. When $\frac{N}{m}$ increases from 8 to 96 bits, $f$ decreases from $2.1 \times 10^{-2}$ to 9.3$\times 10^{-21}$. Obviously, $f$ determines the security strength of our design. For example, when $\frac{N}{m} = 92$ bits, the adversary has to generate around $2^{92.8}$ public/private key pairs on average before finding a valid one to pass the Bloom filter. This is almost computationally infeasible,
at least within the lifetime of the WSN (usually at most several years). However, when $\frac{m}{N} = 64$ bits, the adversary is now expected to generate around $2^{44.4}$ public/private key pairs before finding a valid pair. The analysis below shows the time and cost of the attack. To generate a public/private key pair in ECDSA-160, a point multiplication operation has to be performed, for which the fastest known implementation speed is $0.21\text{ms}$ through a specialized FPGA design [14]. Suppose the adversary could afford 100,000 such FPGAs, which would cost less than one million dollars. Then, by executing 100,000 FPGAs simultaneously, to generate one valid key pair still takes 13.2 hours roughly. With the above analysis, we suggest to select the value of $f$ carefully according to the security requirements of the different types of applications. Given a highly security sensitive military application, we suggest that $f$ should be no larger than $6.36 \times 10^{-20}$, i.e., $m/N \geq 92$ bits. On the other hand, when the targeted applications are less security sensitive as in the civilian scenario, we can tolerate a larger $f$. This is because the adversary is now generally much less resourceful as compared to the former case.

C. Maximum Number of Network Users Supported

It is important to know how many network users can be supported in BAS so that the WSN can be well planned. The following theorem provides the answer.

**Theorem 2:** Given the storage space $m$ bits for a single Bloom filter and the required probability of a false positive $f_{\text{req}}$ ($f_{\text{req}} \in (0, 1)$), the maximum number of network users that can be supported is $N = \frac{m(\ln 2)^2}{\ln f_{\text{req}}}$, that is,

$$N \leq \frac{-0.4805m}{\ln f_{\text{req}}}.$$ 

**Proof:** Since the minimal probability of a false positive $f = 2^{-k}$ is achieved with $k = \frac{m}{N} \ln 2$, we have $f_{\text{req}} = 2^{-\frac{m}{N} \ln 2}$. Then, we can easily get $N = \frac{-m(\ln 2)^2}{\ln f_{\text{req}}}$ in this case; and this is the maximum number of users that can be supported given $f_{\text{req}}$ and $m$. □

Fig. 4 illustrates the maximum supported number of network users as a function of the storage limit. Fig. 4 shows that BAS supports up to 1,250 users when $f_{\text{req}} = 4.42 \times 10^{-14}$, 1,000 users when $f_{\text{req}} = 2.03 \times 10^{-17}$, and 869 users when $f_{\text{req}} = 6.36 \times 10^{-20}$, for a storage space of 9.8 KB. Obviously, BAS also allows tradeoff between the maximum supported number of network users and the probability of a false positive given a fixed storage limit.

D. Supporting More Users using the Merkle Hash Tree: The Hybrid Authentication Scheme (HAS)

Through the above analysis, we know that the maximum supported number of network users is usually limited given the storage limit and the probability of a false positive. For example, if $f_{\text{req}} = 6.36 \times 10^{-20}$ and the storage limit is 4.9 KB, the maximum number of users supported by BAS is 434. Therefore, an additional mechanism has to be employed to support more users when necessary. HAS achieves this goal by employing the Merkle hash tree technique, which trades the message length for the storage space. That is, by increasing the per message overhead, HAS can support more network users. Specifically, HAS works as follows.

The sink first calculates the maximum number of users supported in case of BAS according to the given storage limit and the desired probability of a false positive. It then collects all the public keys of the current network users and constructs a Merkle hash tree. In fact, the sink constructs $N$ leaves with each leaf corresponding to a current user of the WSN. For our problem, each leaf node contains the binding between the corresponding user ID and his public key, that is, $h(U_{ID}, PK_{UID})$. The values of the internal nodes are determined by the method introduced in Section II.C. The sink further prunes the Merkle hash tree into a set of equal-sized smaller trees. We denote the value of the root node of a small hash tree as $h^i_r$, $i = 1, \ldots, |S|$, where $|S|$ equals the maximum number of supported users the sink calculates in BAS.

Next, the sink constructs a Bloom filter $V$ following the same way as described in the last section. The difference is that now the member set $S = \{h^1_r, h^2_r, \ldots, h^{|S|}_r\}$. Then, the sink preloads each sensor node with $V$. At the same time, each user should obtain its AAI according to his corresponding leaf node’s location in the smaller Merkle hash tree. Let $T$ denote all the nodes along the path from a leaf node to the root (not including the root), and $A$ be the set of nodes corresponding to the siblings of the nodes in $T$. Then, AAI further corresponds to the
values associated with the nodes in $A$. Obviously, $AAI$ is of size $(L + \log_2 N)$ bytes, where $L$ is the length of the hash values. Upon user revocation, the sink simply updates all the sensor nodes with the ID information of the revoked users. And each node directly stores the revoked IDs as described earlier. Now a message sent by a user $U_{ID}$ is of form

$$< M_2, C, D, W_{pub}, AAI_{U_{ID}} > . \ (IV)$$

Each node verifies the authenticity of a user public key in two steps. First, it calculates the corresponding root node value $h_1^*$ using $AAI_{U_{ID}}$ attached in the message. Second, it checks whether or not the calculated $h_1^*$ is a member of $V$ stored by itself. By checking Message $(IV)$, we can easily find that HAS doubles the maximum supported number of users as compared to BAS at the cost of 20 more bytes per message overhead, assuming SHA-1 is used [25]. And the number can be further doubled with 40 more bytes per message overhead.

VI. PERFORMANCE ANALYSIS

A. Communication Overhead

We study how the message size affects the energy consumption in communication in a WSN. We investigate the energy consumption as the function of the size of the WSN (denoted as $W$). We denote by $E_{hr}$ the hop-wise energy consumption for transmitting and receiving one byte. As reported in [31], a Chipcon CC1000 radio used in Crossbow MICA2DOT motes consumes 28.6 and 59.2 $\mu$J to receive and transmit one byte, respectively, at an effective data rate of 12.4 Kbps. Furthermore, we assume a packet size of 41 bytes, 32 bytes for the payload and 9 bytes for the header [31]. The header, ensuring an 8-byte preamble, consists of source, destination, length, packet ID, CRC, and a control byte [31]. We also assume that $|M| = 20$ bytes.

Then, for BAS, the signature size is still the same as that of ECDSA, but only part of the message now has to be transmitted, with the saving of up to 10 bytes. Therefore, the message overhead of BAS is 54 bytes, which is 10 bytes less than that of DAS. As Message $(III)$ is 74 bytes, there should be 3 packets in total, among which two of them are 41 bytes, and one is 19 bytes. Therefore, there should be $41 + 19 + 1 + 8 + 3 = 125$ bytes for transmission (including 8-byte preamble per packet). Hence, the hop-wise energy consumption of message transmission is $125 \times 59.2 \mu$J = 7.40 mJ; and the energy consumption of message reception is $125 \times 28.6 \mu$J = 3.58 mJ. For each message broadcast, every sensor node should retransmit the message once and receive $w'$ times of the same message assuming the blind flooding is used. Here, $w'$ denotes node density in terms of the total number of sensor nodes within one unit disc, where a unit disc is a circle area with radius equal to the transmission range of sensor nodes. Hence, the total energy consumption in communication will be $W * (7.4 + 3.58 + w') \text{ mJ}$.

Fig. 5 illustrates the energy consumption in communication as a function of $W$ with $w' = 20$. Clearly, BAS consumes a much lower energy as compared to others. For example, when $W = 15,000$, CAS always costs 2.20 KJ, while BAS costs only 1.18 KJ. The energy saving for a single broadcast can be more than 1,000 J between BAS and CAS. Note that although DAS also consumes much less energy than CAS, DAS only supports up to 10000/22 ~ 454 users. At the same time, BAS can handle 869 users even when $f_{req} = 6.36 \times 10^{-20}$. CAS handles more users than BAS and DAS, however, at the cost of much higher energy consumption. Moreover, HAS can handle a large number of users but with a much lower energy consumption when compared to CAS. In summary, BAS demonstrates the highest communication efficiency, as well as a desirable storage efficiency. From Fig. 5, we also find that the energy consumption in communication is the critical cost for WSNs, as a single broadcast of a message of only 20 bytes in length could cost energy on the order of KJ. This also exposes the severe vulnerability of the $\mu$TESLA-like schemes, as they allow the adversary to flood the WSN arbitrarily.

\*In an idealized lossy network, blind flooding, i.e., every node always retransmits exactly once every unique message it receives, is wasteful, as individual nodes are likely to receive the same broadcast multiple times. In practice, however, blind flooding is a commonly used technique, as its inherent redundancy provides some protection from unreliable (lossy) wireless networks [21].

\*We assume an uniform transmission range for all sensor nodes.
B. Computational Overhead

It was previously widely held that PKC is not suitable in WSNs, as sensor nodes are extremely computation constrained. However, recent studies [8], [31] showed that PKC with only software implementations, is very viable on sensor nodes. For example in [31], an ECC signature verification takes 1.61s with 160-bit keys on ATmega128 8MHz processor used in a Crossbow mote. We analyze the computation cost of the proposed schemes to further justify the suitability of PKC-based schemes in WSNs. In all our proposed schemes, the major computational cost is due to the signature verification operation. In the following analysis we omit the cost of other operations such as hash operations and table lookup, as they are negligible as compared to the signature verification operation [31].

In CAS, two ECDSA signature verifications are needed for each broadcast message. In BAS, to verify a message takes \( k = \frac{m}{N} \ln 2 \) hash operations and one ECDSA signature verification. It was reported in [31] that an ECDSA-160 signature verification operation costs 45.09 mJ on a 8-bit ATmega128L processor running at 4 MHz. If we assume that the sensor CPU is a low-power high-performance 32-bit Intel PXA255 processor, the energy cost can be further minimized. Note that the PXA255 has been widely used in many sensor products such as Sensoria WINS 3.0 and Crossbow Stargate running at 400 MHz. According to [13], the typical power consumption of PXA255 in active and idle modes are 411 and 121 mW, respectively. It was reported in [4] that it takes 92.4 ms to verify an ECDSA-160 signature with the similar parameters on a 32-bit ARM microprocessor at 80 MHz. Therefore, the same computation on PXA255 roughly needs \( 80/400 \times 92.4 \approx 18.48 \) ms, and the energy cost is hence around 7.6 mJ. Therefore, we can obtain the computational costs of the proposed CAS and BAS schemes on different sensor platforms\(^6\). The results are summarized below.

<table>
<thead>
<tr>
<th>Scheme</th>
<th>ATmega128L</th>
<th>PXA255</th>
</tr>
</thead>
<tbody>
<tr>
<td>CAS</td>
<td>90.18 mJ</td>
<td>15.4 mJ</td>
</tr>
<tr>
<td>BAS</td>
<td>45.09 mJ</td>
<td>7.6 mJ</td>
</tr>
</tbody>
</table>

BAS is obviously also more computationally efficient than CAS. Furthermore, when we compare the computational cost with the communication cost on hop-wise message transmission, we can find that both are on the same order, which justifies the suitability of PKC-based schemes in WSNs.

C. Security Strength

The Bloom filter based public key verification ensures the security strength of the proposed scheme by enabling immediate message authentication. That is, there is no authentication delay on messages being broadcast. Therefore, it is very hard for the adversary to perform network wide flooding in the WSN. As we analyzed above, by appropriately choosing a suitable value of \( f_{req} \), such as \( 6.36 \times 10^{-20} \) in military applications, it is infeasible to forge a valid public/private key pair during the lifetime of the WSN. Furthermore, by embedding a time stamp into the message, the message replay attack is also effectively prevented, as WSN is assumed to be loosely synchronized [27]. Therefore, the immediate message authentication capability provided by the proposed schemes can effectively protect the WSN from network wide flooding attacks. This is the most significant security strength over the \( \mu \)TESLA-like schemes, in which network wide flooding attacks are always possible.

Moreover, since the public key operation is expensive, it is also important that sensor nodes can be resistant to the local jamming attacks. Under such attacks, the adversary may simply broadcast random bit strings to the sensor nodes within its transmission range. If these neighbor sensors have to perform the expensive signature verification operation for all received messages, it will be a heavy burden on them. CAS obviously suffers from this type of attacks, as the signature verification operation has to be performed for every received message. However, in both BAS and HAS, such an attack can be effectively mitigated. This is because in both schemes, a sensor node first verifies the authenticity of the attached user public key through hash operations, so it performs signature verification operation for a bogus public key only with a negligible probability (e.g., \( 6.36 \times 10^{-20} \)). As reported in [31], the energy cost of SHA-1 is only 5.9 \( \mu \)J/byte on a 8-bit ATmega128L processor, while ECDSA-160 could consume 45.09 mJ on signature verification. An adversary may also flood the sensor nodes with forged messages but containing valid user public keys, which can be obtained by eavesdropping the network traffic. In this case, the forged messages can only be discarded after signature verification, and sensor nodes that are physically close to the adversary can thus be abused. We note that this type of attacks is always possible for PKC-based security mechanisms. However, this attack can still be mitigated in BAS by implementing an alert

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\(^6\)DAS and HAS consume similar amount of energy as BAS does, as they both require one signature verification.
report mechanism. If a sensor node fails to authenticate the received messages multiple times in a row, it will derive that an attack is going on and alert the sink about the attack. The sink further carries out field investigations or other means to detect the adversary and take corresponding remedy actions that are outside the scope of this paper.

VII. CONCLUDING REMARKS

In this paper, we studied the problem of multiuser broadcast authentication in WSNs. We pointed out that symmetric-key-based solutions such as μTESLA are insufficient for this problem by identifying a serious security vulnerability inherent to these schemes: the delayed authentication of the messages can easily lead to severe energy-depletion DoS attacks. We then came up with several effective PKC-based schemes to address the problem. Both computational and communication costs of the schemes are minimized through a novel integration of several cryptographic techniques. A quantitative energy consumption analysis, as well security strength analysis were further given in detail, demonstrating the effectiveness and efficiency of the proposed schemes.

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